Vortex Rossby Waves and Hurricane Intensiﬁcation in a Barotropic Model

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ABSTRACT

In this study the balanced evolution of small but ﬁnite as well as large-amplitude asymmetries in a rapidly rotating hurricane-like vortex is investigated. In particular, the wave kinematics and wave–mean ﬂow interaction of vortex Rossby waves in a barotropic nonlinear asymmetric balance (AB) model are examined. By diagnosing the evolution of different asymmetric initial potential vorticity (PV) disturbances and their effect on the symmetric vortex, recent linear and quasi-linear predictions are veriﬁed and the proposed AB model is shown to be a viable balance model for azimuthal wavenumbers $l > 1$. For disturbance amplitudes that are 40% of the basic-state PV at the radius of maximum wind, a discrete normal mode propagating cyclonically around the vortex is excited as a by-product of the process by which energy is transferred from the asymmetries into the basic state (axisymmetrization). In addition we are able to show that even a strong disturbance axisymmetrizes in a circular ﬂow and is able to intensify the basic state. Side-by-side comparison with some experiments from a primitive equation model show good agreement for both weak and strong asymmetric disturbances. The results raise intriguing questions about the dynamical role of discrete and continuous spectrum vortex Rossby waves in the moist convective dynamics of the hurricane. The application of the results to hurricane intensiﬁcation will be addressed.

1. Introduction

To a ﬁrst approximation, a hurricane is a symmetric vortex circulation (Ooyama 1982). However, stationary and propagating asymmetric features are known to be ubiquitous features that have an impact on the hurricane (Willoughby et al. 1984; Franklin et al. 1993). The precise manner in which asymmetries, either in the environment or in the near-core region, inﬂuence the hurricane circulation is not yet well understood. In particular, it is not known under what circumstances asymmetric features enhance the storm circulation and under what circumstances they lead to vortex weakening. Here we use a barotropic shallow-water formulation where explicit convection is excluded. Although much of the observed asymmetries are convective in nature, we focus here on the horizontal advective dynamics. Convection is represented to the extent that the prescribed initial potential vorticity (PV) anomalies could be convectively forced. Recognizing that convection does not simply create PV anomalies but rather redistributes PV vertically,¹ a process that cannot be represented in a barotropic model, the present work is intended to be a step toward understanding the mechanism by which asymmetries modulate the development of hurricane-like vortices.

A vortex intensiﬁcation mechanism by convectively forced vortex Rossby waves was proposed by Montgomery and Kallenbach (1997, henceforth MK97). The mechanism is analogous to the mechanism by which planetary Rossby waves can intensify and maintain a large-scale zonal jet (Shepherd 1987) except here the radial PV gradient of the hurricane vortex plays the role of the planetary vorticity gradient. Shepherd’s work shows that the intensiﬁcation mechanism, in particular whether the basic state strengthens or weakens, depends on the strength of nonlinear, quasi-two-dimensional turbulence effects. We will show in the last part of section 3 that at least for the ﬁnite amplitudes considered relevant for hurricane vortices the basic-state vortex ﬂow strengthens at the expense of the induced asymmetry, whether the asymmetry is weak or strong. The present work veriﬁes not only the linear and quasi-linear predictions of MK97 with a nonlinear semispectral shallow-water asymmetric balance (AB) formulation based on Shapiro and Montgomery (1993, hereafter SM93),

¹ Although, as demonstrated by Haynes and McIntyre (1987), the total integrated PV between two isentropic surfaces does not change.
but also shows that a similar mechanism operates when nonlinear effects are strong. Results from the AB formulation are confirmed with a primitive equation (PE) model.

The AB theory is a close relative of the balance equations (McWilliams 1985) but also is formally valid in flow regimes whose asymmetric divergence is not small (Montgomery and Franklin 1998). The validity of the AB theory depends formally on a nondimensional parameter, which can be interpreted as the square of the ratio between the orbital and inertial frequencies in the symmetric vortex. The square of this so-called local Rossby number, $R_n^2$, where $n$ is the azimuthal wavenumber of the asymmetry, is assumed to be $\ll 1$ (see section 2 of SM93). A naive scaling of the orbital frequency for mature hurricane vortices allows $R_n^2 \ll 1$ only for azimuthal wavenumber 1. In practice, however, we will demonstrate why the AB theory nevertheless makes qualitatively correct predictions for higher wavenumber disturbances. The model is initialized with weak-but finite and strong-amplitude PV asymmetries and is integrated over a period of 72 h. During this time the asymmetries transfer most of their energy to the symmetric vortex, and their kinematics and wave–mean flow interaction are usefully characterized by the vortex Rossby wave mechanics of MK97.

2. Nonlinear asymmetric balance model

The balance model is the same as used in MK97 (their section 3b), but the fully nonlinear terms are included based on section 5 of SM93. In case of shear and motion experiments (Møller and Jones 1998) it was sufficient to generalize the AB system by including only the quasigeostrophic nonlinear terms. In this work, however, we wish to investigate the interaction between small-but finite amplitude as well as large amplitude asymmetries and the circular basic-state vortex in the near-core region. In order to examine such problems the AB formulation must be extended to a nonlinear form that incorporates full inertial effects. Referring to SM93’s Fig. 1, observations suggest that the asymmetries of a hurricane above the boundary layer are weak compared to the azimuthal mean vortex. We are aware that there may exist episodic events when the asymmetries are not weak compared to the mean vortex. For example, frictional convergence and convection tend to concentrate and intensify vertical vorticity near the radius of maximum wind (RMW), which through barotropic or baroclinic instability may produce episodes of extensive mixing near and inside the hurricane eyewall. Nevertheless, direct numerical simulations suggest that the ultimate end state of the unforced flow approaches a circular vortex with small-amplitude asymmetries (W. Schubert 1997, personal communication). Thus, as a basis for understanding PV mixing/redistribution processes associated with finite amplitude vortex Rossby waves in hurricanes, we consider first the problem of small but finite amplitude asymmetric PV disturbances on an initially circular basic-state vortex in gradient wind balance.

The proposed nonlinear AB model reduces to the quasigeostrophic shallow-water model as the standard Rossby number tends to zero, and in the presence of mass and/or momentum sources reduces to Eliassen’s balance model in the axisymmetric case. For small-amplitude (linear) asymmetric dynamics the model possesses Wenzel–Kramers–Brillouin (WKB) solutions that are qualitatively consistent with a linear nondivergent model.

Denoting $r$ and $\lambda$ as radius and azimuthal angle, and the initial basic-state tangential wind by $\bar{v}$, the material derivative operator following the basic-state wind is defined as

$$\frac{D_v}{Dt} = \frac{\partial}{\partial t} + \frac{v}{r} \frac{\partial}{\partial \lambda}.$$

![Fig. 1](image-url) (a) Area-integrated pseudo-energy (ordinate) as a function of wavenumber (abscissa) for wavenumber truncation 8 after 2 h (solid line), 4 h (dashed line), and 12 h (dotted line) in $m^2 s^{-2}$ for case 2. (b) Area-integrated potential enstrophy (ordinate) as a function of wavenumber (abscissa) for wavenumber truncation 8 after 2 h (solid line), 4 h (dashed line), and 12 h (dotted line) in $s^2 m^{-2}$ for case 2.
The formulation of the nonlinear AB model begins with radial and azimuthal momentum equations in an earth-based coordinate system expressed in terms of asymmetries superposed on a circular basic-state vortex in gradient wind balance:

\[
\frac{Dv}{Dt} u' - \bar{\xi} v' = -\frac{1}{r} \frac{\partial \phi'}{\partial \lambda} - N'_v, \quad (2.1)
\]

\[
\frac{Dv}{Dt} v' + \bar{\eta} u' = -\frac{1}{r} \frac{\partial \phi'}{\partial \lambda} - N'_u, \quad (2.2)
\]

where \((\phi', u', v')\) denote the initial perturbation geopotential, radial velocity, and tangential velocity, respectively; \(\bar{\xi}\) is the Coriolis parameter; \(\gamma = r^{-1} d(r \bar{\sigma})/dr\) the basic state relative vorticity; \(\bar{\eta} = f + \bar{\zeta}\) the basic-state absolute vorticity; and \(\bar{\zeta} = f + 2\beta/r\) the modified Coriolis parameter. Here \(N'_u\) and \(N'_v\) are, respectively,

\[
N'_u = u' \frac{\partial u'}{\partial r} + v' \frac{\partial u'}{\partial \lambda} - \frac{u'^2}{r},
\]

and

\[
N'_v = u' \frac{\partial v'}{\partial r} + v' \frac{\partial v'}{\partial \lambda} + u' v' - \bar{\xi} v' - \bar{\eta} u'.
\]

Note that primed quantities include the instantaneous azimuthal wavenumber-0 (symmetric) component, generated through the nonlinear interactions represented by \(N'_u\) and \(N'_v\).

In analogy with quasigeostrophic theory, a simple and physically plausible equation set for the asymmetric slow manifold within a rapidly rotating, inertially (centrifugally) stable, gravitationally subcritical (Froude number <1) vortex flow is obtained by evaluating the time derivatives and \(N'_u\) and \(N'_v\) in Eqs. (2.1) and (2.2) with the pseudomomenta \(u'_p = -(\bar{\eta} - \bar{\zeta})^{-1} \partial \phi' / \partial \lambda\) and \(v'_p = \xi^{-1} \partial \phi' / \partial r\) (see section 4 of SM93) in place of the perturbation radial wind \(u'\) and perturbation tangential wind \(v'\). Equations (2.1) and (2.2) are then solved for \(u'\) and \(v'\) and substituted into the continuity equation

\[
\frac{Dv}{Dt} \phi' + \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial (ru')}{\partial r} + \frac{1}{\bar{\eta}} \frac{\partial (v^2)}{\partial \lambda} \right) + u' \frac{\partial \phi'}{\partial r} = 0, \quad (2.3)
\]

yielding a first-order nonlinear evolution equation for the disturbance geopotential \(\phi'\). Here \(\bar{\sigma}\) denotes the instantaneous azimuthal mean geopotential \((= \bar{\sigma} + \bar{\zeta})\), where \(\bar{\sigma}\) is the initial mean height field in gradient balance with \(\bar{\nu}\), where the undisturbed fluid depth is taken to be 1 km in all experiments except where noted. Updated values of the azimuthal mean tangential wind \(\bar{\nu}\) (and hence relative vorticity \(\xi\), modified Coriolis parameter \(\xi\), and potential vorticity \(q = \bar{\sigma}/\bar{\nu}\)) are obtained from the gradient balance equation \(\bar{\sigma} + \bar{\nu}^2/r = \partial \bar{\sigma}/\partial r\). Explicitly, the nonlinear equation for \(\phi'\) is

\[
\frac{D^2 \phi'}{Dt^2} + \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial (ru')}{\partial r} + \frac{1}{\bar{\eta}} \frac{\partial (v^2)}{\partial \lambda} \right) + \left( -1 - \frac{1}{\bar{\eta}} \right) \frac{\partial \phi'}{\partial r} + \left( - \frac{1}{r} \right) \frac{\partial \phi'}{\partial \lambda} + \left( - \frac{1}{\bar{\eta}} N'_u \right) \frac{\partial \phi'}{\partial r} + \left( - \frac{1}{\bar{\eta}} \right) \frac{\partial \phi'}{\partial \lambda} = 0, \quad (2.4)
\]

where

\[
u' = \frac{1}{\bar{\sigma}} \frac{\partial \phi'}{\partial r} - \frac{1}{\bar{\eta}} \frac{\partial \phi'}{\partial \lambda} + \frac{1}{\bar{\eta}} N'_u,
\]

\[\gamma = \frac{1}{\bar{\sigma}} \frac{\partial \phi'}{\partial r} - \frac{1}{\bar{\eta}} \frac{\partial \phi'}{\partial \lambda} + \frac{1}{\bar{\eta}} N'_u,\]

and \(\gamma = \bar{\sigma} / \bar{\nu}\) is the inverse square of the local Rossby deformation radius. Numerical details can be found in appendix A.

3. Experiments

a. Axisymmetrization

On a PV monopole in which the basic-state PV decreases monotonically with radius, linear asymmetries

transfer their energy into the basic state, a process called “axisymmetrization,” due to the differential rotation of fluid around the vortex center (Kallenbach and Montgomery 1995; Sutyrin 1989). The axisymmetrization process is dependent both on radius and on azimuthal wavenumber (Smith and Montgomery 1995). As demonstrated by MK97, the axisymmetrization process is usefully described by vortex Rossby waves, which eventually propagate outward before their symmetrization. MK97 were able to relate these waves to intensity changes in hurricane-like vortices. In this work we test the predictions of MK97 in a nonlinear balance dynamics context and compare our numerical results with their linear and quasi-linear findings.

For all experiments shown here we use the benchmark vortex as described in MK97 (their Fig. 1), which is an initially cyclonic vortex with a maximum tangential
wind $\bar{v} = 36.8 \text{ m s}^{-1}$ at a radius of 70 km. The domain size is 3000-km radius. For the weak-amplitude initializations (sections 3a and 3c) the model is initialized with an azimuthal wavenumber 1, 2, or 3 PV disturbance centered at the RMW with radial distribution $\alpha \sin^2[\pi (r - 20 \text{ km})/(100 \text{ km})]$, where $\alpha$ is the amplitude of the disturbance. Although in principle the initial asymmetry could have any orientation with respect to the horizontal shear, current radar observations offer no indication that upshear orientations are preferred on the mesoscale (MK97, section 3a). We therefore focus on the symmetrization (or decay) phase and examine the evolution of asymmetries that are initially upright with respect to horizontal shear. The PV is given by the perturbation pseudo-PV

$$q' = \frac{\xi}{\phi} - \frac{\partial }{\partial \phi} \frac{b'}{\phi},$$

(3.1)

where the pseudovorticity $\xi' = \mathbf{k} \cdot \nabla \times \mathbf{u}'$ is derived from the pseudomomenta based on the inertia parameter $\xi$:

$$\mathbf{u}' = \left( \frac{-1 \partial \phi'}{r \xi} \frac{\partial \lambda'}{\lambda} \frac{\partial b'}{\partial r} \right).$$

(3.2)

The amplitude of the PV asymmetry is 20% (cases 1, 2, and 3, corresponding to wavenumbers 1, 2, and 3) or 40% (case 4) of the basic-state PV at the RMW. In section 3d a strong initial asymmetry in the form of a single cluster is used. The integration time for all experiments is 72 h. After this period of time the asymmetries have transferred most of their energy into the basic state and therefore are nearly axisymmetrized around the center of the vortex.

In order to allow wave–wave and wave–mean flow interactions we have to include enough wavenumbers. Energy and enstrophy spectra as a function of azimuthal wavenumber for our weak-amplitude experiments show that for low-wavenumber initial conditions (wavenumbers 1–3) a truncation at azimuthal wavenumber 4 is sufficient. Figure 1 illustrates this fact for the initialization of a wavenumber-2 asymmetry (case 2 in section 3b), which shows the time evolution of the domain-integrated pseudoenergy and pseudoenstrophy as a function of azimuthal wavenumber, for truncation at wavenumber 8. The energy is the perturbation pseudoenergy for the shallow-water system, derived in an analogous way to Eq. (4.5) of SM93. Here the pseudoenergy is $(\delta / 2 g)(u'' u'''' + v'' v''') + (\phi^{'''} / 2 g)$, where $g$ is the gravitational acceleration and $u''$ and $v'''$, are defined in an analogous manner to the pseudomomenta in section 2. The potential enstrophy is $\delta / 2 g$ times the square of the pseudo-PV given in Eq. (3.1).

As the enstrophy have spectral peaks at wavenumber 2 that decrease in time. The energy and enstrophy in wavenumbers greater than 4 have their highest amplitude in the first 2 h, much smaller than the wavenumber-2 peak, and also decrease in time. Thus, there is no evidence of any piling up of energy or enstrophy in wavenumbers >4. In the case of the strong asymmetry the system was truncated at azimuthal wavenumber 16; tests with truncation at wavenumber 32 gave virtually identical results.

**b. Initial asymmetry 20% of basic-state vortex**

1) **WAVENUMBERS 1, 2, AND 3**

For the first experiment (case 1) we initialize the model with a wavenumber-1 asymmetry. Similar to the analytical solution of MK97 we see a fast growing mode inside the RMW, which is not a normal mode but rather corresponds to a displaced basic-state vortex center and is therefore called a “pseudomode.” In contrast to MK97, our initialized asymmetry is centered at the RMW, whereas MK97’s asymmetry was at radius $r = 3$, far outside of their RMW ($=1$). As in MK97 we observe outward-propagating vortex Rossby waves. The amplitude of the PV asymmetry decays in time and propagates outward in waves. After about 48 h the asymmetry is mostly axisymmetrized while the pseudomode attains an equilibrium, with its maximum PV amplitude occurring at 40-km radius. The maximum amplitude of the pseudomode is about half the size of the initialized wavenumber-1 asymmetry after 24 h of integration time (Fig. 2) and stays so for the rest of the integration. In MK97’s calculation the pseudomode after time $t = 40$ (corresponding to 24 h) was about half the size of its value after 12 h and then remained about the same.

Case 2, with an initial wavenumber-2 asymmetry, induces an inner-core asymmetry at about 50 km inside the RMW that after 2 h attains a larger amplitude than

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2 Although an inner-core asymmetry also appears in case 1, it is overshadowed by the fast growing pseudomode.
the initial asymmetry itself (Fig. 3). Similarly, MK97 centered their initial asymmetry at their RMW and also found an induced inner-core asymmetry in case 1. In order to explain the induced asymmetry in the center, note that the wavenumber-2 initial condition produces a deformation-like flow across the center of the vortex. In order to conserve total PV the inward-(outward)-moving fluid parcel must acquire a negative (positive) PV perturbation upshear (downshear) and inside (outside) of the initial negative (positive) asymmetry. This so-called leading spiral can grow temporarily at the expense of the basic state before decaying as a trailing spiral due to radial shear. After 2 h the initial asymmetry in case 2 is about half its initial amplitude and propagates outward. After about 4 h the asymmetries, the primary and induced one, have a similar size and propagate outward. The maximum amplitude of the wavenumber-2 asymmetry moves to about 120-km radius after 24 h and is more than one order of magnitude smaller than the initial value (Fig. 3). The stagnation radius, where the maximum PV amplitude is situated, is slightly outside of the location where the tangential wind tendency is zero (cf. Fig. 6a), consistent with MK97’s section 2h. Apart from the presence of a finite deformation radius, these results are qualitatively consistent with what one finds using wave-activity diagnostics in a quasi-linear nondivergent formulation (e.g., Held and Phillips 1987). During the remainder of the integration time, shear-enhanced diffusive damping (see note added in proof in MK97, p. 446) controls the decay of the PV asymmetry.

Case 3, with an initial wavenumber-3 asymmetry, induces less of a response due to the wavenumber-dependent scale of influence (cf. MK97, section 2e). The amplitude of the primary asymmetry decreases after 2 h to half of its initial size, after 12 h it is about one order of magnitude smaller, and after 24 h two orders of magnitude smaller than its initial value (not shown). As the wavenumber increases the radial group velocity decreases (MK97) and the asymmetry is sheared away before it can propagate and transport energy outward.

Figure 4 shows horizontal contour plots of the asymmetric PV. In order to compare the different spatial extents of all three cases we specify the same contour interval. In case 1, the asymmetries circulate cyclonically around the vortex center and radiate outward. Already after about 6 h the outward-propagating features reach 400 km. After 24 h the outer contours extend to 600 km, whereas in the center the pseudomode attains a constant amplitude. After 72 h the outer edges of the asymmetries extend to 800 km (Fig. 4). Although the results are qualitatively similar to MK97, the radial propagation in our simulation is more apparent as the asymmetry starts at a much smaller radius.

In case 2 and 3, there is no pseudomode and it is easier to follow the asymmetries. Again the asymmetries circulate cyclonically around the vortex center (cf. right-hand side panels of Fig. 5 in MK97) and first propagate inward and outward, and then only outward. In case 2 (case 3) the asymmetries in the center start to propagate outward after 20 h (16 h) (Figs. 5a,b). After 32 h (24 h) the asymmetries are axisymmetrized in the center and inside a radius of 100 km they essentially disappear (Figs. 5c,d). After 72 h the asymmetries propagate farther out and essentially disappear inside a radius of 150 km. In contrast to wavenumber 1 the outer edges of the asymmetries extend to only 500-km radius for case 2 (Fig. 5e), whereas for case 3 by this time the asymmetry essentially disappears (being below the contour interval of Fig. 5). The results of cases 1–3 are consistent with the WKB prediction of MK97, showing that the higher the wavenumber the more the asymmetry is radially confined. This can be explained readily by the dependence of the radial group velocity on azimuthal wavenumbers [cf. Eq. (17) of MK97]. As the azimuthal wavenumber increases, the radial group velocity decreases and the shearing effect dominates over the radial propagation.

2) Wave–mean flow interaction

We now investigate the wave–mean flow interaction by calculating the change in the azimuthally averaged tangential velocity ($\overline{v}$), PV ($\overline{q}$), vorticity ($\overline{\zeta}$), and geopotential height ($\overline{\phi}$) of the basic state. In the first 2 h there is for case 1 an initial weakening of $\overline{v}$ of about 0.25 m s$^{-1}$ near the center out to 70-km radius, due to growth of the inner-core asymmetry [not shown; see explanation for wavenumber 2 in section 3b(1), second paragraph]. This local weakening of the vortex decreases after 4 h to about 0.1 m s$^{-1}$ for the remainder of the integration time as the induced leading spiral became a trailing spiral due to the anticyclonic shearing of the mean tangential wind. Outside of the RMW the vortex

Fig. 3. Amplitude of the wavenumber-2 PV asymmetry ($10^{-4}$ s m$^{-1}$) for case 2, where the initial amplitude equals 20% of $\overline{q}$, initially (dotted line), after 2 h (dashed line), and after 24 h (solid line).
spins up within a few hours to about 0.2 m s\(^{-1}\) (also not shown). In case 2 the deceleration of the tangential velocity inside the RMW is much weaker than in the former case and stays overall constant. This suggests that the upshear-induced asymmetry is weaker than in case 1 but remains approximately steady. The acceleration of the tangential velocity has a maximum of 0.19 m s\(^{-1}\) just outside of the RMW, and the maximum deceleration is 0.13 m s\(^{-1}\) at 115-km radius (Fig. 6a). Although these changes are very small, one should bear in mind that the vortex is forced here by an initial PV asymmetry only. As convection is not an initial forcing but an ongoing process, one can expect that multiple reinitialization of the forcing would increase the acceleration and lead to an intensification of the vortex. A study of Montgomery and Enagonio (1998) shows that their change of the tangential velocity of 1 m s\(^{-1}\) increased by about a factor of 10 with a simulation of five “pulses” of convective activity in the form of PV anomalies added at intervals of 0.5 eddy turnover times (which is for our vortex about 1.5 h) to the total PV field. Whereas for the cases described above the acceleration is always stronger than the deceleration, the acceleration and deceleration in case 3 are about the same. In that case the deceleration has a maximum of 0.14 m s\(^{-1}\) and the acceleration of about 0.13 m s\(^{-1}\) (not shown). The maxima are also located just outside the RMW and at 120-km radius. Inside a radius of 50 km there is nearly no change of the basic tangential velocity.

In order to confirm our results from the balanced AB model we compare the wave–mean flow interaction for case 2 with a PE model. The PE shallow-water model is essentially the same as that used in Shapiro and Ooyama (1990), but with higher horizontal resolution chosen to match that of the AB model, and is initialized with the same vortex and asymmetry. After 6 h the change of the basic-state tangential velocity (Fig. 6a; L. Shapiro 1998, personal communication) is very similar to the one in our experiment (cf. Fig. 6a). The maximum acceleration is 0.19 m s\(^{-1}\) just outside the RMW as in our case, and the maximum deceleration is 0.11 m s\(^{-1}\) 10 km inside the radius in the AB model.

The acceleration and deceleration of the basic-state vortex and their relative locations can be explained by

![Fig. 4. Horizontal contourplot of the wavenumber-1 PV asymmetry (initially 20% of \(q\); case 1), after (a) 6, (b) 24, (c) 72 h. Contour interval \(1 \times 10^{-11}\) s m\(^{-2}\).](image-url)
Fig. 5. Horizontal contourplot of (a) wavenumber-2 PV asymmetry after 20 h, (b) wavenumber-3 PV asymmetry after 16 h, (c) wavenumber-2 PV asymmetry after 32 h, (d) wavenumber-3 PV asymmetry after 24 h, and (e) wavenumber-2 PV asymmetry after 72 h. Contour interval $1 \times 10^{-11} \text{ s m}^{-1}$. Initial amplitude equals 20% of $\bar{q}$. 
a simple thought experiment using rectilinear (zonal)
constant shear flow. The meridional shear is assumed
positive. If a localized positive PV anomaly is super-
imposed on this constant shear flow the anomaly will
tilt downshear in time. The induced eddy momentum
fluxes will be maximized at the center of the anomaly,
thereby accelerating the zonal flow to the north of the
anomaly and decelerating it to the south. The analogous
process in a vortex will accelerate the tangential wind
radially inward of where the deceleration occurs. In the
symmetrization process the energy of the asymmetry
goes into the basic state and angular momentum will be
redistributed, giving different maximum values of the
acceleration and deceleration.

Case 1 gives a different result in the PV change than
cases 2 and 3. In the first 2 h the PV decreases inside
a radius of 50 km, eventually leaving only very weak
small-scale (∼10 km) radial variations. The PV increas-
es between 50- and 90-km radius, associated with an
upgradient PV flux. Between 90 and 130 km the basic-
state PV decreases, associated with a downgradient PV
flux. In case 2 (case 3) the PV increases between 40
and 80 km, decreases between 80 and 120 km (80 and
110 km), and increases again between 120 and 210 km
(110 and 180 km) (case 2 is shown in Fig. 6b; case 3,
not shown). The radial variations are smooth for both
cases. The reduction of the PV accelerates in time for
both cases and is even stronger in case 3. The more
confined wavenumber-3 asymmetry induces a stronger
change in the mean PV, possibly due to the localized
character of the disturbance. The change of the basic-
state vorticity (Fig. 6c) is similar to the change in the
basic-state PV. The change in the geopotential height is
consistent with gradient wind balance. The correspond-
ing changes in $\theta$, $\zeta$, and $\Phi$ in the PE model (see Figs.
6b–d) are consistent with the change in $\Psi$ (Fig. 6a). Note,
in particular, that the change in $\Phi$ in the PE model is
$\sim$1 m$^2$ s$^{-2}$ less than that in the AB model inside ∼100-
kam radius, consistent with the stronger deceleration in
the AB model outside that radius (Fig. 6a). The decrease
of the geopotential height in the inner-core region cor-
responds to a lowering of isentropes and therefore a
stabilization in that region, whereas the raising of is-
entropes outside this region corresponds to destabili-

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**Fig. 6.** (a) Change of the basic-state tangential velocity (in m s$^{-1}$) after 6 h, calculated with the AB theory (solid line) and calculated with the PE model (dashed line); case 2, where the initial amplitude equals 20% of $\Psi$. (b) As in Fig. 6a but for change of the basic-state potential vorticity (10$^{-9}$ s m$^{-2}$) after 6 h. (c) As in Fig. 6a but for change of the basic-state vorticity (10$^{-5}$ s$^{-1}$) after 6 h. (d) As in Fig. 6a but for change of the basic-state geopotential (m$^2$ s$^{-2}$) after 6 h.
Fig. 7. Amplitude of the wavenumber-2 PV asymmetry ($10^{-8}$ s m$^{-2}$) initially (dotted line), after 2 h (dashed line), and after 24 h (solid line) for case 4, where the initial amplitude equals 40% of $\bar{q}$.

*Fig. 8. Horizontal contourplot of the wavenumber-2 PV asymmetry (initially 40% of $\bar{q}$; case 4), after (a) 12, (b) 24, and (c) 72 h. Contour interval $2 \times 10^{-11}$ s m$^{-2}$."

c. Initial asymmetry 40% of basic-state PV, a wave-induced unstable vortex

1) Wavenumber 2

In this experiment we initialize the vortex with a wavenumber-2 PV asymmetry (Fig. 7), where the amplitude is now 40% of the basic-state PV (case 4). Similar to case 2 an inner-core asymmetry is induced. Here...
the primary and induced asymmetry have similar size and are half the size of the initial asymmetry after about 2 h of integration time. In contrast to case 2 where the induced asymmetry propagated outward, the induced asymmetry in case 4 stays for at least 24 h inside a radius of about 100 km, before its amplitude starts to decrease (Fig. 7). After 72 h the induced asymmetry inside the RMW persists (in contrast to case 2; cf. Fig. 3) but is much weaker than initially (2 orders smaller). Near 130-km radius the asymmetry has small-scale (~10 km) radial variations and after 72 h a peak is left at that radius.

The horizontal contourplots of the PV asymmetries (Fig. 8) give us a very different result, compared with the experiments discussed before. Since the amplitude of the initial condition is twice that of the earlier cases the contour interval is twice as large as in cases 1, 2 and 3. In the first 10 h the behavior is quite similar. The asymmetries rotate cyclonically around the vortex center and propagate first inward and outward, and then outward. In this simulation, however, not all of the wavenumber-2 asymmetries are axisymmetrized away. After 12 h, a wavenumber-2 asymmetry stays in the center, decreases, and then increases again (Fig. 8). In order to understand the differences in the axisymmetrization process in case 4, which seems to be less complete than in the previous cases, we take a closer look at the radial gradient of the PV. The mean PV of the initial vortex used here decreases monotonically in radius (Fig. 9a), which thus satisfies Rayleigh’s sufficient condition for stability (Gent and McWilliams 1986; Montgomery and Shapiro 1995). After 4 h of integration time, however, the radial gradient of the PV, \( \frac{\partial q}{\partial r} \), changes sign at about 110-km radius and after 6 h also at 150-km radius (Fig. 9b). The size of the positive gradient of PV has a maximum at 130-km radius and the vortex is potentially unstable between 6 and 36 h. During this time a quasi-neutral wavenumber-2 discrete normal mode becomes established from the center out to 100-km radius. Unlike the continuous spectrum of vortex Rossby waves, this discrete vortex Rossby wave is able to maintain its structure despite the presence of differential rotation. After
about 36 h the PV gradient becomes negative again and the normal mode slowly decays.

Similar results are found for initialization with a wavenumber-3 asymmetry. In the case of wavenumber 1 we were not able to identify a normal mode. Although numerical solutions are no substitute for a proof that nonstationary wavenumber-1 modes do not exist, our numerical results are nevertheless consistent with Gent and McWilliams (1986) who found no discrete mode for $n = 1$ other than the pseudo-mode despite strong sign reversals in all basic-state radial vorticity gradients.

2) WAVE–MEAN FLOW INTERACTION

As the initialized wavenumber-2 PV asymmetry is twice as large in this experiment here compared with case 2, one might expect a response of the basic-state vortex that is also twice as large. However, as the wave–mean flow interactions are nonlinear the feedback is more complicated, as seen in the change of $\overline{\tau}$ and $\overline{q}$ (Fig. 10). The main features look very similar to case 2. The maximum of the acceleration of the tangential velocity is just outside the RMW, and between 110 and 250 km $\overline{\tau}$ decelerates. Already after 2 h the increase of $\overline{\tau}$ is about 0.6 m s$^{-1}$, and the decrease 0.4 m s$^{-1}$. The increase in $\overline{\tau}$ reaches 0.8 m s$^{-1}$, and the decrease 0.6 m s$^{-1}$. Inside a radius of 60 km, there is a weak decrease of $\overline{\tau}$ of about 0.02 m s$^{-1}$, which stays constant during the 72 h (Fig. 10a). The change of $\overline{\tau}$ is similar to $\overline{\tau}$, about four times stronger than in case 2, consistent with the quadratic nature of the interaction between the asymmetries and the vortex. The radial location of the increase/decrease in $\overline{\tau}$ is also similar to case 2; here $\overline{\tau}$ increases between 50- and 80-km, decreases between 80- and 130-km, and increases again between 130- and 230-km radius (Fig. 10b).

$\overline{q}$

d. Strong asymmetry in the form of a single-cluster convective anomaly

In order to test the applicability of the former experiments with weak asymmetries to a case with a strong
asymmetry, here we initialize the vortex with a PV disturbance in the form of a single cluster. The strength of the PV asymmetry, centered at about 115-km radius (Figs. 11a,b), is more than half of the magnitude of the basic-state PV maximum and 200% of the PV at the RMW. As in the experiments before, the single-cluster asymmetry circulates cyclonically around the center and radiates outward. After 6 h the maximum of the asymmetry is about halved (Fig. 11c), decreases in time (Fig. 11d), and after 24 h the asymmetry is about 10% of the magnitude of its initial value (Fig. 11e). After 72 h the single-cluster anomaly is nearly axisymmetrized (not shown). Consistent with the axisymmetrization process, the change in the basic-state tangential velocity results in a net intensification of the vortex. Figure 12 shows the change of $\overline{\sigma}$ after 6 h. The maximum acceleration is 0.7 m s$^{-2}$ at 90-km radius, and the deceleration is 0.3 m s$^{-2}$ at 140-km radius. If one would carry out the same experiment in a pulsing mode, as mentioned in section 3b(2), this could lead to a substantial intensification of the vortex. In order to assess the accuracy of the balance simulation for such a high-amplitude initial condition the same experiment is carried out with the PE model mentioned in section 3b(2). Similar to our results $\overline{\sigma}$ accelerates by 0.6 m s$^{-2}$ at 100-km radius, and decelerates by 0.2 m s$^{-2}$ at 140-km radius (Fig. 12; L. Shapiro 1998, personal communication). Although the AB theory is derived for small-amplitude disturbances and is stretched for this experiment, the PE and AB results are not only qualitatively but even quantitatively similar [though the differences are somewhat larger than those in section 3b(2) for case 2].

4. Conclusions

In this study we have investigated the vortex Rossby wave–mean flow dynamics of rapidly rotating barotropic vortices using the AB model. Specifically, we examined the axisymmetrization process of different disturbances. We initialized the model with small but finite amplitude wavenumber-1, -2, and -3 PV asymmetries, as well as a strong single-cluster asymmetry, and studied the corresponding change of the basic-state vortex. In the case of an initial wavenumber-2 PV asymmetry that was 20% of the basic-state PV we found an acceleration inside and a deceleration outside the RMW. These results were confirmed with a PE model. When the asymmetry was 40% of the basic-state PV we found in addition a wave-induced eigenmode that interacts with the vortex. At even higher wave amplitudes ($\sim$200% of the basic-state PV at the RMW) the PV asymmetries in both AB and PE still axisymmetrize and strengthen the basic-state vortex.

Although the AB theory is formally valid for wave-number 1 only, the nonlinear AB formulation proposed here validates the self-consistency of this theory for small as well as strong-amplitude asymmetric disturbances and suggests its usefulness for higher wave-number disturbances. In order to understand why the results of the AB model are so similar to the PE model given that the naive scaling of the squared local Rossby number implies large inaccuracies for wavenumbers >1, appendix B takes a closer look at the scaling. As an example, the squared local Rossby number for the case-2 experiment was calculated in two ways for azimuthal wavenumber 2 (the dominant asymmetry): first the naive $R_n^2$ and then the complete $D^2$ applied to the solution for $u'_n$ (see appendix B). Figure 13a shows $D^2 u'_n /Dt^2$, $(n^2 \overline{\sigma}^2/r^2)u'_n$, and $\overline{\sigma} u''_n$, where the complete numerator of the squared local Rossby number, $D^2 /Dt^2$, is about five times smaller than the naive one, $(n^2 \overline{\sigma}^2/r^2)u'_n$. Figure 13b shows $D^2$ and $R_n^2$ only out to a radius of 200 km, since the amplitudes farther out (see Fig. 13a) approach zero asymptotically. As expected, the naive scaling $R_n^2$ substantially overestimates the squared local Rossby number, and the calculated $D^2$ is about five times smaller. Similar results were obtained for $u'_n$. This overestimate explains why the AB theory does well even for $n > 1$.

With respect to real hurricanes, one can imagine that convection creates asymmetries similar to the ones used in this paper. The asymmetries are able to change the vortex structure, but the stronger asymmetry (40% of $\overline{\sigma}$) changes the vortex insofar that the vortex becomes slightly unstable and is able to support a discrete eigenmode. The vortex could sustain the eigenmode, which then itself could interact with the convection and then feed back to the vortex. All of our experiments forced the vortex only by an initial asymmetry, resulting in a weak response. As mentioned in section 3b, cumulus convection is in reality not a single but an ongoing process, which could be simulated by multiple reinitialization of the forcing (pulsing experiments), which could then lead to a substantial vortex intensification. The effect of such pulsing, which is a subject of ongoing work, is beyond the scope of the present investigation. The present barotropic results are one step toward understanding the basic dynamics of hurricanes and trough interactions. The ongoing research in three dimensions will examine the precise manner in which either environmental or near-core region convectively induced disturbances influence the hurricane circulation.

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APPENDIX A

Numerical Details

The numerical model used to solve Eq. (2.4) is semispectral, with Fourier modes in the azimuthal coordinate
and grid points in the radial coordinate. The nonlinear terms are computed by explicitly summing a convolution sum in Fourier space. Second-order centered differences are employed to approximate radial derivatives. Inversion for the geopotential tendency $\partial \phi / \partial t$ is accomplished with a tridiagonal solver. Denoting the wavenumber component by a subscript and the Fourier transform by a caret, the boundary conditions are then as follows: For wavenumbers $n \geq 1$, $\partial \phi / \partial t = 0$ at $r = 0$ and $r = r_{\text{max}}$ (=3000 km, outer boundary); while for $n = 0$, $\partial^2 \phi / \partial r^2 \partial t = 0$ at $r = 0$ and $\partial \phi / \partial r \partial t = 0$ at $r = r_{\text{max}}$.

The numerical model is time stepped with a fourth-order Runge–Kutta scheme with typical time increments equal to 5 min. With a radial grid spacing of 2.5 km, the chosen time step falls below the empirically determined CFL stability threshold. In the cases of the weak asymmetries (sections 3b and 3c) $\delta t = 5$ min; in the case of the strong single-cluster anomaly (section 3d) $\delta t = 30$ s. All runs use 1200 radial grid points and employ explicit diffusion of $\phi^2$ so as to remove finescale PV associated with the potential enstrophy cascade. The explicit form of the diffusion term added to the right-hand side of Eq. (2.4) is $\nu \nabla^2 \phi^2$, which has the desirable property of ensuring that the contribution to the perturbation energy is negative definite in integral. We choose $\nu = 1.0 \times 10^{13}$ m$^4$ s$^{-1}$. When $\nu$ is multiplied by $\gamma^2$, which is held fixed at its value at $r = 0$, this corresponds to an effective second-order diffusivity of 233 m$^2$ s$^{-1}$. The effect of diffusion on the local and global PV and energy conservation is found to be negligible. Setting the diffusion to zero in the experiment in section 3c gave essentially the same results. We find that the total integrated PV and energy in each experiment, including the strong asymmetry, changes negligibly during its duration ($<10^{-2}$% for PV, and $<10^{-3}$% for energy for the strong asymmetry, and smaller for the weak-asymmetry cases).

**APPENDIX B**

Local Rossby Number

As mentioned in the introduction, the square of the local Rossby number is the ratio between the orbital acceleration and the inertial stability. In the naive scaling SM93 approximated the orbital acceleration $D^2 /Dr^2$ [where $D^2 /Dr^2 = \delta^2 /\delta t^2 + (\nabla \omega) \delta /\delta \lambda$] by the advective contribution so that $D^2 /Dr^2 \sim n^2 \nabla^2 \ell /r^2$. Then the ratio between the orbital acceleration and the inertial stability is the squared local Rossby number $R^2 = (n^2 \nabla^2 \ell /r^2) / (\eta \xi)$. The AB theory is derived by neglecting terms $O(R^2)$, which increase as the square of $n$ and so becomes very large for high azimuthal wavenumber. In practice, however, there may be a cancelation between the local time rate of change and the advective term, so that $D^2 /Dr^2 \sim n^2 \nabla^2 \ell /r^2$. In that case the naive scaling will substantially overestimate the squared local Rossby number.

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